BI-TP 2005/21

J/ψ photo- and electroproduction, the saturation scale and the gluon structure function*

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Abstract

Our approach to low-x deep inelastic scattering based on the $\vec{l}_\perp$ factorization of perturbative QCD (the color-dipole picture) yields parameter-free absolute predictions for J/ψ production. The connection of J/ψ production to the low-x saturation scale and to the gluon structure function is clarified.

* Supported by Deutsche Forschungsgemeinschaft, contract number schi 189/6-2 and the Ministry of Education and Science, Japan under the Grant-in-Aid for basic research program B (no. 17340085)

Vector meson photoproduction and electroproduction provide a significant test of the theory of inclusive deep-inelastic scattering (DIS) at low values of the Bjorken scaling variable, $x \simeq Q^2/W^2 \ll 1$. In the present paper, we confront our theoretical results [1] on DIS and vector-meson production with recently published experimental data on J/ψ production [2].

The low-x kinematics, within QCD, implies $\vec{l}_\perp$ factorization[1] or, equivalently, the color-dipole picture [3]. The persistence of the two-gluon-exchange structure[2] of the γ^*-proton forward-scattering amplitude allows one to incorporate low values of Q^2, including $Q^2 = 0$, into a unified description of the photoabsorption cross section at low x and all Q^2 [4]. This point of view is supported by the empirical evidence for low-x scaling [5, 4], $\sigma_{\gamma^* p} = \sigma_{\gamma^* p}(\eta(W^2, Q^2))$, that says that large and small values of Q^2 yield identical photoabsorption cross sections, once the corresponding energies are appropriately chosen to imply identical values of the scaling variable $\eta(W^2, Q^2)$ that is explicitly defined below.

At low x, in the color-dipole picture from QCD, one may explicitly represent the Compton-forward-scattering amplitude in terms of forward scattering of $(q\bar{q})^{J=1}$ (vector) states [6]. In refs. [4, 6], we made the simplifying assumption that the forward-scattering amplitude is independent of whether the $(q\bar{q})^{J=1}$ states have transverse or longitudinal polarization. In the dual language of parton distributions, in the kinematic domain where appropriate, this simplifying assumption turned out to be equivalent to an underlying proportionality of the sea-quark and the gluon distribution [7], i.e.

$$(q\bar{q})_{\rm sea} \quad \sim \quad \alpha_s(Q^2) \cdot \text{gluon distribution} \quad \sim \quad \Lambda^2_{sat}(W^2 = Q^2/x). \tag{1}$$

The proportionality (1) together with the assumed power-like increase of the "saturation scale", $\Lambda^2_{sat}(W^2)$, as a function of the energy, W,

$$\Lambda^2_{sat}(W^2) \sim (W^2)^{C_2}, \tag{2}$$

upon requiring consistency with DGLAP evolution, led to the remarkable conclusion that the value of C_2 must be fixed at [7]

$$C_2^{theory} = 0.276 \tag{3}$$

in agreement with the previous fit [4] to the experimental data,

$$C_2^{exp} = 0.27 \pm 0.01. \tag{4}$$

[1]Here $\vec{l}_\perp$ stands for the transverse momentum of the gluon.

[2]Note that the structure of the amplitude, i.e. the expression for the color-dipole cross section in transverse position space is dictated by the gauge-invariant coupling of the two gluons to the $q\bar{q}$ pair. This structure has to survive the transition to the "soft" domain of $Q^2 \to 0$.

The W-dependence of $\Lambda^2_{sat}(W^2)$ determines the approach to saturation in the sense of

$$\lim_{\substack{W^2\to\infty \\ Q^2 \text{fixed}}} \frac{\sigma_{\gamma^* p}\left(\eta(W^2,Q^2)\right)}{\sigma_{\gamma p}(W^2)} = 1, \tag{5}$$

or[3]

$$\lim_{\substack{W^2\to\infty \\ Q^2 \text{fixed}}} 4\pi^2\alpha \frac{F_2(x,Q^2)}{\sigma_{\gamma p}(W^2)} = Q^2 \tag{6}$$

i.e. the approach to the "soft" energy dependence of the total photoproduction cross section at any fixed value of $Q^2 > 0$. The scaling variable in (5) is given by

$$\eta(W^2,Q^2) = \frac{Q^2 + m_0^2}{\Lambda^2_{sat}(W^2)}. \tag{7}$$

The mass $m_0^2 < m_\rho^2$ (where m_ρ denotes the ρ-meson mass) enters via quark-hadron duality [10, 11].

The saturation scale, $\Lambda^2_{sat}(W^2)$, specifies the effective transverse momentum of the gluons coupled to the $q\bar{q}$ pair in the two-gluon exchange amplitude, [4]

$$\langle \vec{l}_\perp^{\,2} \rangle_{W^2} = \frac{1}{6}\Lambda^2_{sat}(W^2). \tag{8}$$

In addition to $\Lambda^2_{sat}(W^2)$, one needs the integral over the transverse gluon distribution, $\sigma^{(\infty)}$, that determines the normalization of the total photoabsorption cross section. The specification of the two integrals over the unintegrated gluon distribution, $\Lambda^2_{sat}(W^2)$ and $\sigma^{(\infty)}$, is sufficient to determine the photoabsorption cross section or the proton structure function at low x for $Q^2 \ll \Lambda^2_{sat}(W^2)$ and $Q^2 \gg \Lambda^2_{sat}(W^2)$, respectively. This is true under the above-mentioned assumption of the equality of transverse and longitudinal $(q\bar{q})^{J=1}$ scattering or property (1). Compare also [8], where the equality is replaced by a proportionality, and the connection with the longitudinal and transverse parts of the proton structure function is elaborated upon. The complete dependence on W and Q^2 for all W and Q^2 at low x not only requires a knowledge of the integrated quantities but an ansatz for the gluon-momentum dependence that specifies how $\Lambda^2_{sat}(W^2)$ and $\sigma^{(\infty)}$ appear in the photoabsorption cross section.

[3] Compare ref. [9] for a plot of the experimental data according to (6).

[4] Relation (8) follows from $\Lambda^2_{sat}(W^2) \equiv \frac{\pi}{\sigma^{(\infty)}}\int d\vec{l}_\perp^{\prime 2}\vec{l}_\perp^{\prime 2}\bar{\sigma}_{(q\bar{q})_L^{J=1}} = \frac{6\pi}{\sigma^{(\infty)}}\int d\vec{l}_\perp^{\,2}\vec{l}_\perp^{\,2}\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2)$, where $\bar{\sigma}_{(q\bar{q})_L^{J=1}}$ follows from $\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2)$ by $J=1$ projection, and $\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2)$ specifies what is frequently called "the unintegrated gluon distribution". The above relation to be valid requires the normalization condition $\frac{\sigma^{(\infty)}}{\pi} = \int d\vec{l}_\perp^{\,2}\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2) = \int d\vec{l}_\perp^{\prime 2}\bar{\sigma}_{(q\bar{q})_L^{J=1}}(\vec{l}_\perp^{\,2}, W^2)$ to hold. The normalization condition must not necessarily hold, since $\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2)$ in general contains higher, $J > 1$, partial wave contributions. The above normalization condition is true for $\tilde{\sigma}(\vec{l}_\perp^{\,2}, W^2) = \frac{\sigma^{(\infty)}}{\pi}\delta(\vec{l}_\perp^{\,2} - \frac{1}{6}\Lambda^2_{sat}(W^2))$, that is the ansatz for the dipole cross section underlying our results for DIS and the present paper on J/Ψ production. Some (mild) deviation in the numerical factor of 1/6 in (8) cannot be strictly excluded purely on general grounds.

Since our representation of the virtual Compton-forward-scattering amplitude explicitly contains the amplitudes for $(q\bar{q})^{J=1}$ forward scattering, the transition to vector-meson production is straight-forward indeed. Taking away the outgoing photon yields the production amplitude for a massive $J = 1$ quark-antiquark continuum. Integration of this continuum over an appropriate mass interval, ΔM_V^2, in the approximation of quark-hadron duality [10][5], then determines the vector-meson-production cross section. For J/ψ production, in particular, we have [1]

$$\frac{d\sigma_{\gamma^* p \to J/\psi\ p}}{dt}(W^2, Q^2)\,|_{t=0} = \int_{\Delta M^2_{J/\psi}} dM^2 \int_{z_-}^{z_+} dz \frac{d\sigma_{\gamma^* p \to (c\bar{c})^{J=1} p}}{dt\ dM^2\ dz}(W^2, Q^2, z, m_c^2, M^2), \tag{9}$$

with

$$z_\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4\frac{m_c^2}{M^2}}, \tag{10}$$

where $M^2 \equiv M^2_{c\bar{c}}$ denotes the mass of the produced $c\bar{c}$ pair. The mass of the charm quark, m_c, enters via the light-cone wave function of the incoming virtual photon that describes the $\gamma^* \to c\bar{c}$ transition. We refer to ref. [1] for the explicit expression of the integrand in (9).

The cross section on the right-hand side in (9) depends on the values of $\sigma^{(\infty)}$ and on the parameters in $\Lambda^2_{sat}(W^2)$ as determined in our analysis of DIS. We have[6] [4, 6]

$$\sigma^{(\infty)} = 48 GeV^{-2} = 18.7 mb, \tag{11}$$

and

$$\begin{aligned} \Lambda^2_{sat}(W^2) &= B\left(\frac{W^2}{W_0^2} + 1\right)^{C_2} \\ &\cong B'\left(\frac{W^2}{1GeV^2}\right)^{C_2}, \end{aligned} \tag{12}$$

where C_2 is given by $C_2 = C_2^{theory}$ in (3) and

$$\begin{aligned} B &= 2.24 \pm 0.43 GeV^2, \\ W_0^2 &= 1081 \pm 124 GeV^2, \end{aligned} \tag{13}$$

as well as

$$B' = 0.340 \pm 0.063 GeV^2. \tag{14}$$

[5]Compare also ref. [12] for a recent application of quark-hadron duality.

[6]Note that the relevant quantity in DIS is $R_{e^+e^-}\sigma^{(\infty)}$. The value of $\sigma^{(\infty)}$ in (11) corresponds to 4 quark flavors.

The high-energy approximation in the second line of (12) is even satisfactory for low values of W^2. There are then essentially only two adjusted quantities to describe DIS at low x, namely $\sigma^{(\infty)}$ and the normalization, B', of the saturation scale. At HERA energies, we approximately have $2GeV^2 \lesssim \Lambda^2_{sat}(W^2) \lesssim 7GeV^2$.

The explicit expression for the cross section in (9) in [1] ignores the finite (longitudinal) momentum transfer ($t_{min} \neq 0$) occurring in the inelastic process of vector meson production (in distinction from DIS), as well as the contribution of the real part of the amplitude to the J/Ψ-forward-production cross section. The finite momentum transfer in the two-gluon-exchange approach implies different longitudinal momenta ($x \neq x'$) of the two gluons. These are absent in the color-dipole or $\vec{\ell}_\perp$-factorization approach. Their effect, called skewness, was analysed in terms of generalized gluon structure functions and can (approximately) be incorporated by a multiplicative factor, i.e. $\sigma^{(\infty)}$ in the expression for the J/Ψ-production in (9) is to be replaced by $\sigma^{(\infty)} R_g(C_2)$ [12] with

$$R_g(C_2) = \frac{2^{2C_2+3}}{\sqrt{\pi}} \frac{\Gamma(C_2 + \frac{5}{2})}{\Gamma(C_2 + 4)}\bigg|_{C_2=0.276} = 1.27, \tag{15}$$

and C_2 being identical to the exponent of the W dependence in (12). The correction for the real part of the production amplitude for a power-law in energy W approximately amounts to a factor of $\sqrt{1+r^2} \cong 1.12$, yielding an increase in the J/Ψ production cross section by about 20% [12]. Altogether we thus have the substitution

$$\sigma^{(\infty)} \to \sigma'^{(\infty)} = \sigma^{(\infty)} \times 1.27 \times 1.12 = 68.3\mathrm{GeV}^{-2} \cong 26.6\ mb, \tag{16}$$

to be applied to the explicit expression for the cross section [1] on the right-hand side in (9).

The J/ψ-production cross section (9) in addition depends on the charm-quark mass

$$m_c = 1.5GeV, \tag{17}$$

and on the integration interval,

$$\Delta M^2_{J/\psi} = 3GeV^2. \tag{18}$$

The integration over dM^2 in (9) then runs over the mass interval from $(2m_c)^2 = 3^2GeV^2 = 9GeV^2$ to $12GeV^2$, where the upper integral boundary corresponds to $\frac{1}{2}(M(\Psi')^2 + M(J/\Psi)^2) \cong 12GeV^2$ with $M(\Psi') = 3.7GeV$ and $M(J/\Psi) = 3.1GeV$.

The results of the experiments [2] were given in terms of the J/ψ production cross section, $\sigma_{\gamma^* p \to J/\psi\ p}(W^2, Q^2)$, and the t distribution that was fitted by an exponential, $exp(-b|t|)$. Rather than attempting to extract the experimental forward production cross section for $t \cong 0$ in (9) from the experimental data, we

multiply the theoretical result (9) including correction (16) by the inverse of the experimentally determined parameter b

$$\sigma_{\gamma^* p \to J/\psi\ p}(W^2, Q^2) = \frac{1}{b} \frac{d\sigma_{\gamma^* p \to J/\psi\ p}}{dt}(W^2, Q^2)\,|_{t=t_{\min}} \tag{19}$$

and compare with the experimental results for $\sigma_{\gamma^* p \to J/\psi\ p}(W^2, Q^2)$. For b we use a value of

$$b = 4.5 GeV^{-2} \tag{20}$$

that is approximately equal to the experimental value for photoproduction at $W = 90 GeV$. For $W \cong 30 GeV$ and $W \cong 300 GeV$, the parameter b decreases and increases, respectively, by approximately 8 % and 17 %. In electroproduction, b decreases by about 10 % to 20 % at the largest available values of Q^2 [13]

In fig. 1 and in fig. 2, we compare the theoretical predictions for the J/ψ-production cross section according to (9) (including corrections for skewing and the real part) and (19) with the experimental results from HERA. The parameters for the theoretical predictions are specified in (11) to (18). There is agreement with experiment for the Q^2 dependence at $W = 90 GeV$ shown in fig. 1, and for the W dependence in photoproduction ($Q^2 = 0$) in fig. 2.

The theoretical predictions in figs. 1 and 2 are based on the constant value of $b = 4.5 GeV^{-2}$ from (20). We have checked that the change of b with Q^2 and with the energy, W, implies changes in the theoretical predictions that are within the error band of the experimental data.

A comment on the reliability of our absolute predictions, in particular on their normalization, may be appropriate. An increase of $\Delta M^2_{J/\Psi} = 3$ GeV^2 in (18) to $\Delta M^2_{J/\Psi} = 4$ GeV^2 or, alternatively, a decrease of $m_c = 1.5$ GeV to $m_c = 1.4$ GeV in (17) implies an increase of the J/Ψ photoproduction cross section by about 25 %. In connection with potential uncertainties of the absolute normalization of the cross section in our approach it is worth noting that other approaches [14] require arbitrary fit factors ranging from 1.33 to 2.17 to achieve agreement with the data for J/Ψ photoproduction.

With respect to an intuitive understanding of the above numerical results from (9), it will be rewarding to examine an approximate evaluation [1] of the cross section (9). The approximation replaces the cross section for the production of the $c\bar{c}$ open charm continuum on the right-hand side in (9) by its value at threshold,

$$\begin{aligned} &\frac{d\sigma_{\gamma^* p \to (c\bar{c})^{J=1} p}}{dt\ dM^2 dz}\ (W^2, Q^2, z, m_c^2, M^2) \to \\ &\qquad \to \frac{d\sigma}{dt\ dM^2\ dz}\left(W^2, Q^2, z = \frac{1}{2}, M^2 = 4m_c^2 = M^2_{J/\psi}\right). \end{aligned} \tag{21}$$

The integral in (9) then reduces to

$$\int_{4m_c^2}^{4m_c^2+\Delta M_{J/\psi}^2} dM^2 \int_{z_-}^{z_+} dz = \int_{4m_c^2}^{4m_c^2+\Delta M_{J/\psi}^2} dM^2 \sqrt{1-\frac{4m_c^2}{M^2}} \equiv \Delta F^2(m_c^2, \Delta M_{J/\psi}^2). \tag{22}$$

For $M_{J/\psi}^2$ the experimental value of $M_{J/\psi}^2 = 3.1^2 GeV^2$ is substituted in (21). It lies above threshold in the interval introduced via quark-hadron duality,

$$4m_c^2 < M_{J/\psi}^2 < 4m_c^2 + \Delta M_{J/\psi}^2. \tag{23}$$

Quark confinement prevents the decay of the J/ψ into free quarks.

With the approximation (21), and introducing the notation from (22), the J/ψ-production cross section (9) with correction (16) becomes

$$\begin{aligned} \frac{d\sigma_{\gamma^* p\to J/\psi\ p}}{dt}(W^2,Q^2)\,|_{t=t_{\min}\cong 0} &= \frac{3}{2}\frac{1}{16\pi}\frac{\alpha\ R^{(J/\psi)}}{3\pi}(\sigma'^{(\infty)})^2 \cdot \\ \cdot\ \frac{\Lambda_{sat}^4(W^2)}{(Q^2+M_{J/\psi}^2)^3}&\frac{1}{\left(1+\frac{\Lambda_{sat}^2(W^2)}{Q^2+M_{J/\psi}^2}\right)^2}\Delta F^2(m_c^2,\Delta M_{J/\psi}^2), \end{aligned} \tag{24}$$

where $R^{(J/\psi)} = 4/3$. Expressing the electroproduction cross section (24) in terms of the photoproduction cross section given by (24) at $Q^2 = 0$,

$$\begin{aligned} \frac{d\sigma_{\gamma p\to J/\psi\ p}}{dt}(W^2,Q^2=0)\,|_{t=t_{\min}\cong 0} &= \frac{3}{2}\frac{1}{16\pi}\frac{\alpha R^{(J/\psi)}}{3\pi}(\sigma'^{(\infty)})^2 \cdot \\ \cdot\frac{\Lambda_{sat}^4(W^2)}{\left(M_{J/\psi}^2\right)^3}&\frac{1}{\left(1+\frac{\Lambda_{sat}^2(W^2)}{M_{J/\psi}^2}\right)^2}\Delta F^2(m_c^2,\Delta M_{J/\psi}^2), \end{aligned} \tag{25}$$

we have

$$\begin{aligned} \frac{d\sigma_{\gamma^* p\to J/\psi\ p}}{dt}(W^2,Q^2)\,|_{t=t_{\min}\cong 0} &= \frac{d\sigma_{\gamma p\to J/\psi\ p}}{dt}(W^2,Q^2=0)\,|_{t=t_{\min}\cong 0}\ \cdot \\ \cdot\frac{M_{J/\psi}^2}{(Q^2+M_{J/\psi}^2)}&\cdot\frac{(M_{J/\psi}^2+\Lambda_{sat}^2(W^2))^2}{\left(Q^2+M_{J/\psi}^2+\Lambda_{sat}^2(W^2)\right)^2}. \end{aligned} \tag{26}$$

We stress that the strong increase (as $\Lambda_{sat}^4(W^2)$) of J/ψ photoproduction in (25) is a unique consequence of the threshold condition, $4m_c^2 = 9GeV^2 \simeq 3.1^2 GeV^2 = 9.6GeV^2$. It has nothing to do with the absolute value of m_c^2 and $M_{J/\Psi}^2$ relative to $Q^2 = 0$. At asymptotic energies, for $\Lambda_{sat}^2(W^2) \gg M_{J/\psi}^2$, the J/ψ photoproduction cross section in (25) becomes energy independent, or at most weakly dependent on energy, if we relax the (approximate) constancy of $\sigma^{(\infty)}$ by allowing for a weak dependence on W.

Since the $c\bar{c}$ mass in the cross section under the integral in (9) appears [1] in the combination of $Q^2+M_{c\bar{c}}^2$, we expect that the accuracy of the approximation of the

J/ψ forward-production cross section given by (24) will improve with increasing Q^2. This is indeed seen in fig. 3. In fig. 3, we also show the result obtained upon multiplication of (24) by an ad-hoc factor (of magnitude 2/3) that normalizes the cross section at $Q^2 = 0$ to the empirical value of the photoproduction cross section. The resulting Q^2 dependence, explicitly given in (26), is consistent with the experimental data.

The theoretical expression for the Q^2 dependence in (26) that follows from (9) upon applying the approximation (21) is of interest with respect to the fit of the Q^2 dependence by the ZEUS and H1 collaborations. Their fit in terms of an ad hoc power-law ansatz,

$$\begin{aligned} &\sigma_{\gamma^* p \to J/\psi\ p}(W^2 = 90^2 GeV^2, Q^2) = \\ &= \sigma_{\gamma p \to J/\psi\ p}(W^2 = 90^2 GeV^2, Q^2 = 0) \frac{\left(M^2_{J/\psi}\right)^n}{\left(Q^2 + M^2_{J/\psi}\right)^n} \end{aligned} \tag{27}$$

gave the result [13]

$$\begin{aligned} n &= 2.486 \pm 0.080 \pm 0.068 \\ &\cong 2.49 \pm 0.15. \end{aligned} \tag{28}$$

The success of this ad hoc fit is understood by comparison with our theoretical result (26). The additive contribution from $\Lambda^2_{sat}(W^2)$ in the denominator of the theoretical expression in (26), in the fit with the ansatz (27) is effectively simulated by the non-integral power of $n = 2.49$ given in (28).

The H1 and ZEUS collaborations, by assuming s-channel helicity conservation, have extracted the ratio $R_{L/T}$ of longitudinal-to-transverse J/Ψ production from their measurements of the J/Ψ density-matrix elements. The proportionality (1) of the sea quark and the gluon distribution or, equivalently, the equality of forward production cross section for transverse and longitudinal polarization, with the approximation (21), implies [1]

$$R_{L/T} = \frac{Q^2}{M^2_{J/\Psi}}. \tag{29}$$

A comparison with the experimental data[15, 13] for $R_{L/T} \cong r^{04}_{00}/(1 - r^{04}_{00})$ shows approximate agreement in the Q^2 dependence with a tendency for the absolute normalization to lie above the experimental result.

We turn to the interpretation of J/ψ production in terms of the gluon-structure function. As a consequence from the duality of the color-dipole picture, or $\vec{l}_\perp$ factorization, and γ^*-gluon scattering, in the diffraction region of $x \ll 1$, and for Q^2 sufficiently large, $Q^2 \gg \Lambda^2_{sat}(W^2)$, we have the identification [16, 4],

$$\alpha_s(Q^2) x g(x, Q^2) = \frac{1}{8\pi^2} \sigma^{(\infty)} \Lambda^2_{sat} \left(W^2 = \frac{Q^2}{x} \right), \tag{30}$$

i.e. the function of x and Q^2 on the lefthand side only depends on W^2, once x is replaced by $x = Q^2/W^2$. The identification (30) holds in the DGLAP region of $Q^2 \gg \Lambda^2_{sat}(W^2)$, where [4, 7]

$$\sigma_{\gamma^* p}(\eta(W^2, Q^2)) \sim \frac{F_2(x, Q^2)}{Q^2} \sim \frac{\Lambda^2_{sat}(W^2)}{Q^2}. \tag{31}$$

We note that the factor $R_g(C_2)$ in (15) which is relevant for J/ψ production is recovered by applying the substitution $x \to 0.41x$ [17] in (30).

The W-dependence of the saturation scale and its consequences with respect to the gluon structure function are a unique result of our approach to DIS at low x. According to (30), a determination of $\Lambda^2_{sat}(W^2)$ by measuring the W-dependence of J/ψ production according to (9) or (24) yields a unique x dependence of the gluon structure function for any chosen fixed value of $Q^2 \gg \Lambda^2_{sat}(W^2)$. Since $\Lambda^2_{sat}(W^2)$ is independent of Q^2, it is irrelevant at what value of Q^2 the energy dependence of J/ψ production is actually measured. A measurement of the energy dependence of e.g. J/ψ photoproduction (at $Q^2 = 0$) yields the x-dependence of the gluon structure function for any fixed $Q^2 \gg \Lambda^2_{sat}(W^2)$ just as well as a measurement of the energy dependence of J/Ψ production at $Q^2 \gg \Lambda^2_{sat}(W^2)$.

Since the identification (30) requires sufficiently large Q^2, a representation of J/ψ production as a function of x and Q^2 in terms of the gluon structure function can only exist for large values of Q^2. Substituting (30) into the large-Q^2 approximation of (24),

$$\begin{aligned} \frac{d\sigma_{\gamma^* p \to J/\psi\ p}}{dt}(W^2, Q^2)\,|_{t=t_{\min}\cong 0} \quad &= \quad \frac{3}{2}\frac{1}{16\pi}\frac{\alpha R^{(J/\psi)}}{3\pi}(\sigma'^{(\infty)^2}) \cdot \\ &\cdot \quad \frac{\Lambda^4_{sat}(W^2)}{\left(Q^2 + M^2_{J/\psi}\right)^3}\Delta F^2\left(m_c^2, \Delta M^2_{J/\psi}\right) \end{aligned} \tag{32}$$

we have

$$\begin{aligned} &\frac{d\sigma_{\gamma^* p \to J/\psi\ p}}{dt}\left(W^2 = \frac{Q^2}{x}, Q^2\right)\Big|_{t=t_{\min}\cong 0} \\ &= \frac{3}{2}\frac{1}{16\pi}\frac{\alpha R^{(J/\psi)}}{3\pi}\left(8\pi^2\right)^2 \cdot \frac{\left(\alpha_s(Q^2) x g(x, Q^2)\right)^2}{\left(Q^2 + M^2_{J/\psi}\right)^3} \cdot R_g^2(C_2)(1 + r^2) \cdot \\ &\quad \Delta F^2\left(m_c^2, \Delta M^2_{J/\psi}\right), \end{aligned} \tag{33}$$

(where $Q^2 \gg \Lambda^2_{sat}(W^2)$)[7].

The notion of the gluon-structure function being used in the DGLAP fits of DIS breaks down for small values of Q^2. For $Q^2 \to 0$ it becomes meaningless to replace $\Lambda^2_{sat}(W^2)$ in the $Q^2 \to 0$ cross section in (25) by the gluon structure

[7]The dependence on $(1/Q^6)(\alpha_s(Q^2) x g(x, Q^2))^2$ in (33) agrees with the one in ref. [18]

function according to (30) with the aim of representing the W-dependence of J/ψ production in terms of the x-distribution of a gluon structure function in the limit of small Q^2, or $Q^2 \to 0$. As mentioned before, this does not prevent one from measuring $\Lambda^2_{sat}(W^2)$ at $Q^2 = 0$ according to (25) and from deducing the gluon structure function at $Q^2 \gg \Lambda^2_{sat}(W^2)$ from such measurements.

An issue closely related to the above discussion concerns the prediction of the energy dependence of J/ψ photoproduction from the gluon-structure function extracted from DIS measurements. According to the identification (30), the measured x-distribution of the gluon-structure function at large Q^2 directly yields the W-dependence of J/ψ photoproduction by substitution of $\Lambda^2_{sat}(W^2)$ into (25).

In fig. 4, in addition to the result from the numerical evaluation of quark-hadron duality from fig. 2, we show the result from the approximation (25) upon normalization by a factor of 3/2, as mentioned before in the discussion to fig. 3. The agreement with the more precise numerical evaluation of (9) and with the experimental results is very good indeed. The fact that the W-dependence in the denominator in (25) is relevant is seen by comparing with the result obtained upon ignoring the denominator in (25). The W-dependence becomes much too steep and it even cannot be repaired by an ad hoc multiplication by a constant factor. We note that ignoring the denominator in (25) is equivalent to incorrectly using the large-Q^2 approximation in (32) and (33) at $Q^2 = 0$. This is of relevance with respect to the usual statement[8] that the production of J/ψ mesons even in photoproduction is given by (33) with the gluon structure function taken at an appropriate scale. This conjecture is not supported by our analysis. For $Q^2 \to 0$ a cross section of the form (25) is relevant, where $\Lambda^2_{sat}(W^2)$ may be identified with the gluon-structure function at large Q^2 according to (30).

Various fits of gluon structure functions from DIS have been used to predict [14] J/ψ photoproduction, some of them being successful after ad hoc adjustments of the normalization by factors between 1.3 and 2.2. From our analysis, two conditions have to be fulfilled for a successful representation of J/ψ photoproduction:

i) The gluon structure function at large Q^2 upon substitution of $x = Q^2/W^2$ has to fulfill (30), at least in good approximation. Otherwise, the right-hand side in (30) will depend on W^2 as well as Q^2, and a scale ambiguity will remain. In such a case no unique conclusion on the W dependence of photoproduction can be obtained, since there is no preferred value of $Q^2 \gg \Lambda_{sat}(W^2)$ to be employed in the prediction for the energy dependence of $Q^2 = 0$ photoproduction.

ii) The $Q^2 \to 0$ cross section must be of the form (25), where for $(\sigma^{(\infty)})^2 \Lambda^4_{sat}(W^2)$ in the numerator the large-Q^2 gluon-structure function according to the proportionality (30) is to be substituted. The form of the cross section

[8] Compare e.g. the talk by Teubner at DIS05 [14]

(25) is a straight-forward consequence of the underlying QCD structure, wherein two gluons of transverse momentum $\vec{l}_\perp$ couple to the $c\bar{c}$ pair, combined with the massive-quark threshold relation, $M^2_{J/\psi} \cong 4m_c^2$. Ignoring the W^2-dependent denominator in (25) corresponds to incorrectly applying the large-Q^2 form (33) at $Q^2 = 0$ by putting $Q^2 = 0$ in the denominator.

In summary, based on the coupling of the $c\bar{c}$ pair to two gluons according to perturbative QCD, with $\sigma^{(\infty)}$ and $\Lambda^2_{sat}(W^2)$ taken from the analysis of the total photoabsorption cross section, we have obtained an absolute prediction of forward J/ψ photo- and electroproduction. The successful application of quark-hadron duality implies that the dependence of the cross section on the wave function of the outgoing J/ψ meson can be neglected. The final results can be put into a very simple form that allows for a transparent understanding of the underlying theoretical ansatz.

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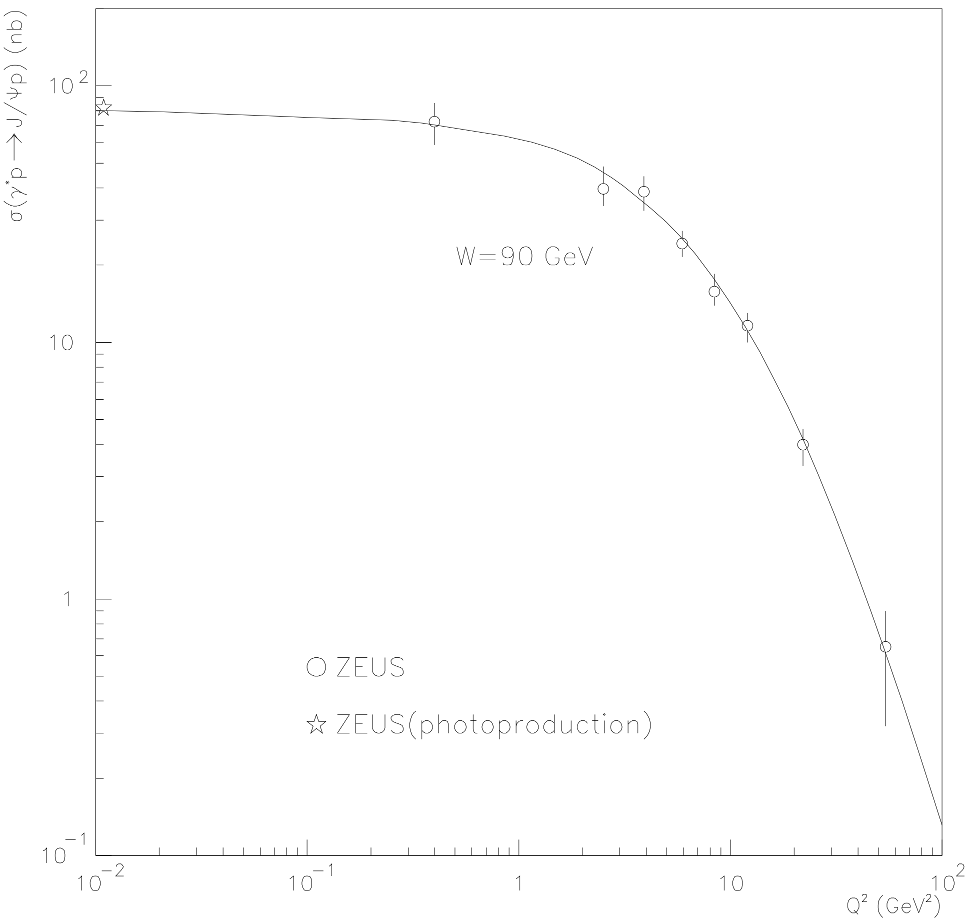

Figure 1: The Q^2-dependence of the cross section for J/ψ production, $\sigma_{\gamma^* p \to J/\psi\ p}(W^2, Q^2)$, at the energy W of $W = 90 GeV$. The theoretical curve is obtained by applying charm-quark hadron duality to $\gamma^* p \to c\bar{c}\ p$ forward production. The experimental data are from the ZEUS collaboration [2].

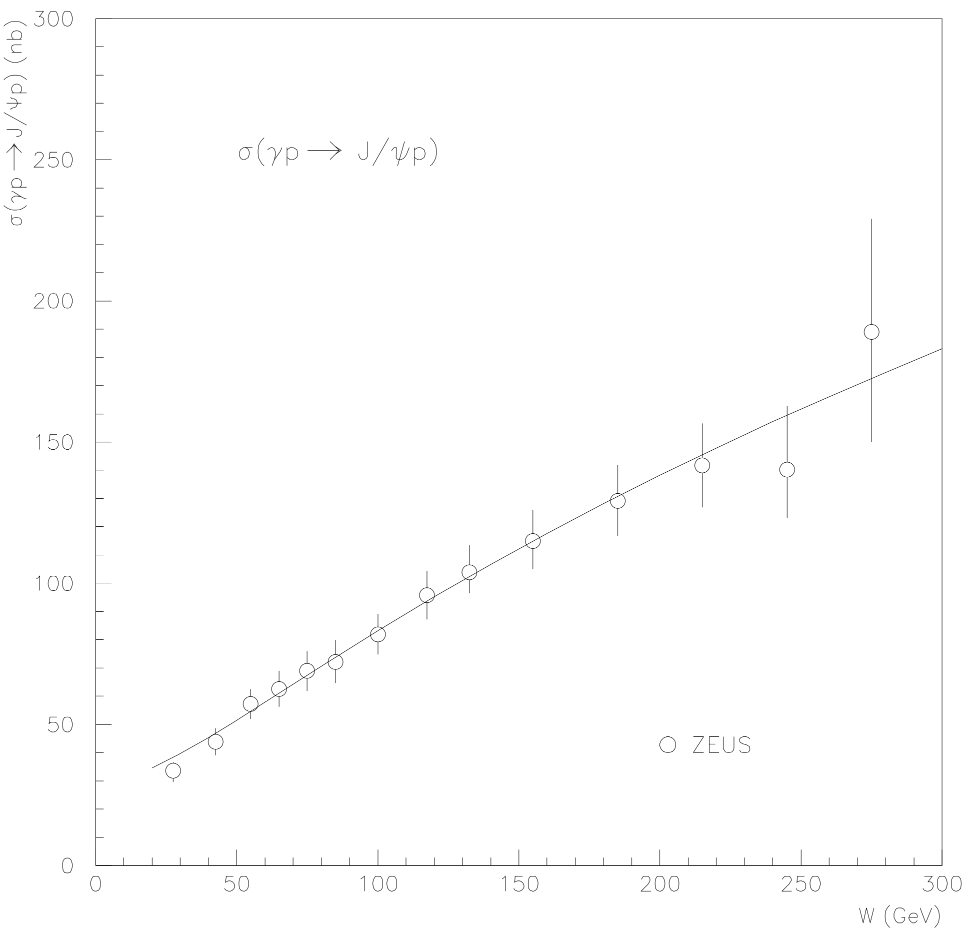

Figure 2: Same as fig. 1, but for the W-dependence of J/ψ photoproduction ($Q^2 = 0$).

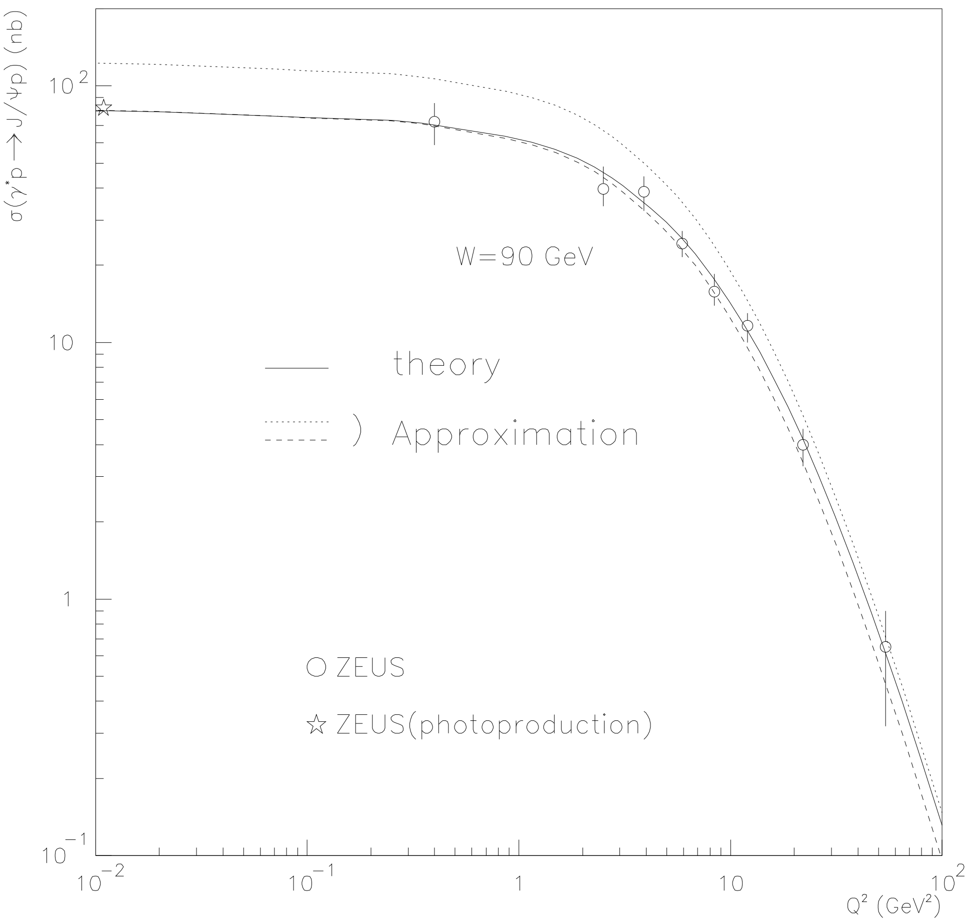

Figure 3: In addition to the results in fig. 1, we show the Q^2-dependence according to (24), obtained by the approximate evaluation of charm-quark hadron duality (dotted curve). The lower (dashed) curve is obtained by normalizing at $Q^2 = 0$ to photoproduction (compare also (26)) by multiplication of the result in (24) and (25) with an appropriate factor.

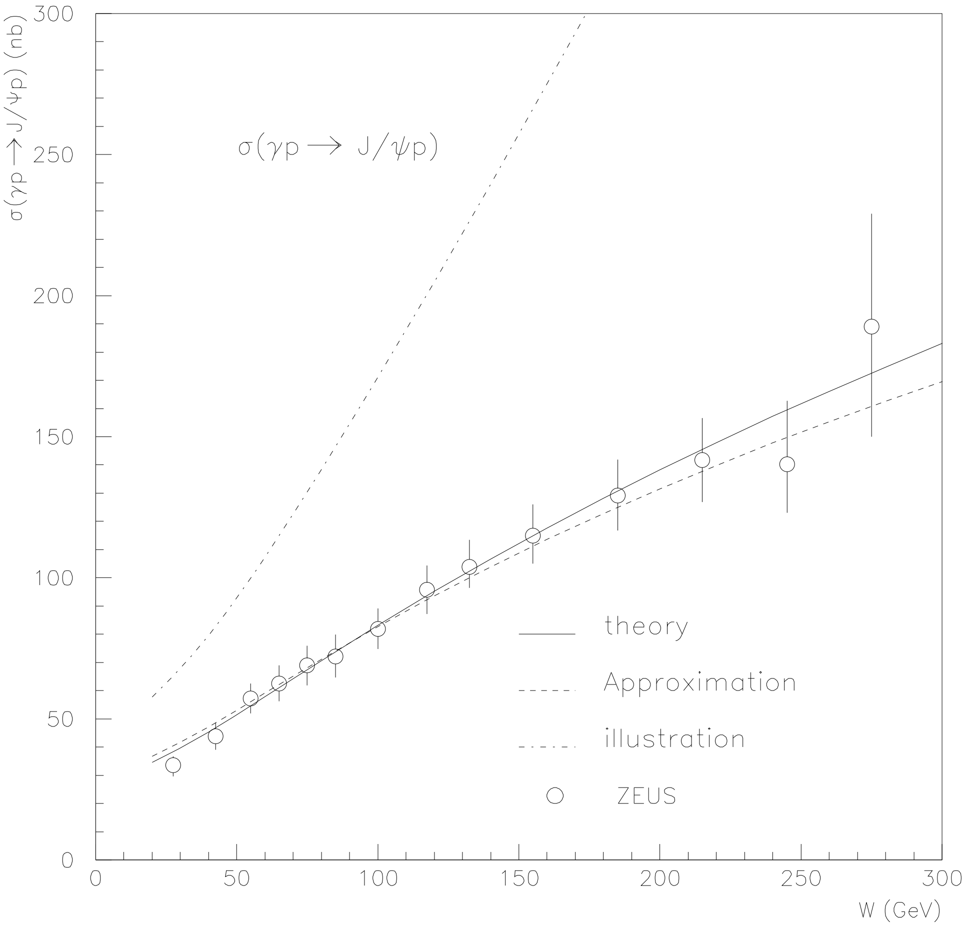

Figure 4: In addition to the results in fig. 2, we show the W-dependence from the approximation (25) normalized to the experimental result at $W = 90GeV$ as in fig. 3 (dashed curve). The dash-dotted curve illustrates what happens, if the large-Q^2 approximation in (32) and (33) is – incorrectly – applied by putting $Q^2 = 0$ in the denominator i.e. by ignoring the W-dependent factor in the denominator of (25).